Math 53: Multivariable Calculus **Sections 201, 204**

Worksheet for 2021-11-22

For each of the following problems:

- Carefully observe what kind of integral it is. For example: "the work done by a vector field along a curve in 2 dimensions," "the integral of a scalar function over a surface in 3 dimensions," etc.
- List all the ways you know of handling such an integral. Then pick one (there might be multiple good approaches).
- Reduce the problem to single or iterated integral(s) with *explicit* upper and lower bounds (i.e. $\int_?^? \int_?^? \cdots$ instead of something like $\iint_S$). Don't compute the integral.

Question 1. $\iint_S \langle -x, -y, z^3 \rangle \cdot d\mathbf{S}$ where S is $z = \sqrt{x^2 + y^2}, 1 \le z \le 3$, oriented downwards.

Question 2. $\int_C (x^2 y^2 \, dx + xy \, dy)$ where C travels in a parabolic arc $y = x^2$ from $(0,0)$ to $(1,1)$, and then in a straight line to $(0,1)$, and then in a straight line back to $(0,0)$.

Question 3. $\iint_S \langle x, y, 5 \rangle \cdot d\mathbf{S}$ where S is the outwards-oriented boundary of the region enclosed by the cylinder $x^2 + z^2 = 1$ and the planes $y = 0$ and $x + y = 2$.

Question 4. $\int_C (\nabla \cdot \langle x, z, zy \rangle) \, ds$ where C is the intersection of $z = 4x^2 + y^2$ and $y = x^2$, inbetween the planes $x = 0$ and $x = 1$.

Question 5. $\iint_S (\nabla \times \langle e^{xy}, e^{xz}, x^2 z \rangle) \cdot d\mathbf{S}$ where S is $4x^2 + y^2 + 4z^2 = 4$, $y \ge 0$, and oriented in the positive y-direction.

Question 6. $\iint_S \langle z, y, xz \rangle \cdot d\mathbf{S}$ where S is the ellipsoid $x^2 + 2y^2 + 3z^2 = 4$, oriented inwards.

Question 7. $\int_C \langle x^2 z, xy^2, z^2 \rangle \cdot d\mathbf{r}$ where C is the ellipse that is the intersection of $x + y + z = 1$ and the cylinder $x^2 + y^2 = 9$, oriented counterclockwise when viewed from above.

Question 8. $\int_C \langle z^2, x^2, y^2 \rangle \cdot d\mathbf{r}$ where C is the line segment from $(1,0,0)$ to $(4,1,2)$.

Question 9. $\iint_S dS$ where S is the portion of the surface $z = xy$ that is contained inside the cylinder $x^2 + y^2 = 1$.

Question 10. $\int_C \langle \frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \rangle \cdot d\mathbf{r}$ where C is the polar curve $r = 2 + \cos(4\theta), 0 \le \theta \le 2\pi$.

Question 11. $\int_C \langle -xy^2, x^2 y, e^{5z} \rangle \cdot d\mathbf{r}$ where C is parametrized by $\mathbf{r}(t) = \langle \cos t, \sin t, e^t \sin^3 t \cos^4 t \rangle$, $0 \le t \le 2\pi$.

Question 12. $\iiint_E dV$ where E is the region enclosed by the surface obtained by rotating the polar curve $r = \cos(2\theta)$, $-\pi/4 \le \theta \le \pi/4$, around the y-axis.